$\Rightarrow 1 = \left(\frac{X}{X_m}\right)^2 + \left(\frac{V}{X_m\,\omega_0^2}\right)^2$

$\Rightarrow X_m^2 = X^2 + \frac{V^2}{\omega_0^2}$

$\Rightarrow \frac{V^2}{\omega_0^2} = X_m^2 - X^2$

$\Rightarrow V^2 - \omega_0^2 (X_m^2 - X^2)$

$\Rightarrow V = \mp\, \omega_0 \sqrt{X_m^2 - X^2}$

$x = 2$ cm

$V = \mp\, 20 \sqrt{(4.10^{-2})^2 - (2.10^{-2})^2}$

$V = \mp\, 0{,}69.\text{m.s}^{-1}$

d- Système (solide, ressort)

$E = E_c + E_p = \frac{1}{2} m V^2 + \frac{1}{2} k x^2$

$\frac{dE}{dt} = \frac{1}{2} m (2va) + \frac{1}{2} k (2xv)$

$= mva + Kxv = v \underbrace{(ma + kx)}_{=0} = 0$

$E = \text{constante}$

$E = \frac{1}{2} k X_m^2 = \frac{1}{2} m V_m^2$

$= \frac{1}{2} 80 (4.10^{-2})^2$

$E = 6{,}4.\ 10^{-2}$ J

2°)

a- $x(t) = X_m \sin(\omega_0 t + \varphi_X)$

$\omega_0 = \sqrt{\frac{k}{m}} = 20.\text{rad. s}^{-1}$

À $t = 0$ $x_0 = X_m \sin(\varphi_X) = 4.10^{-2}$m

$V_0 = X_m\, \omega_0 \cos\varphi_x = 0{,}6\ \text{m.s}^{-1}$

$$\begin{cases} \sin\varphi(x) = \frac{x_0}{X_m} \\ \cos\varphi(x) = \frac{V_0}{X_m\,\omega_0} \end{cases}$$

$1 = \frac{x_0^2}{X_m^2} + \frac{v_0^2}{X_m^2\,\omega_0^2} \Rightarrow X_m^2 = x_0^2 + \frac{v_0^2}{\omega_0^2}$

$X_m = \sqrt{x_0^2 + \frac{v_0^2}{\omega_0^2}} = 5.10^{-2}$m

$\sin(\varphi_x) = \frac{X_0}{X_m} = \frac{4}{5} = 0{,}8$

$\varphi_X = 0{,}92$ rad. car $\cos\varphi_X$ □ 0

$x(t) = 5.\ 10^{-2} \sin(20t + 0{,}92)$ m

$V_{max} = X_m \,.\, \omega_0 = 5.\ 10^{-2}.20 = 1\ \text{m.s}^{-1}$

$x = 2$ cm

$V = \mp\, 20 \sqrt{(5.\ 10^{-2})^2 - (2.\ 10^{-2})^2} = \mp\, 0{,}91\ \text{m.s}^{-1}$

b- $E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} m V_m^2 = \frac{1}{2} m V_m^2$

$E = \frac{1}{2} k x_0^2 + \frac{1}{2} m v_0^2 = 0{,}1$ J